3

Unit

Fundamentals of Atomic Theory

Neils Bohr
(1885 – 1962)

Learning Objectives:

After studying this chapter, students should be able to:

- Understand the concepts of black body radiation and photoelectric effect.
- Describe the atomic models and limitations.
- Describe the Bohr's atomic model and calculate the velocity of electron, radius of orbit and n^{th} state energy of Hydrogen atom.
- Interpretation of the Hydrogen spectrum, different spectral series and energy level diagram of H-atom.
- Know about the terms, emission spectra, absorption spectra and critical potential.
- Describe the experimental determination of critical potential (Franck-Hertz experiment).
- Know about dual nature of radiation, de-Broglie's theory (hypothesis).
- Explain the experimental verification of de-Broglie's theory.
- Understand uncertainty principle and describe its applications.
- Describe the physical origin or uncertainty principle.
- Know about the matter waves and uncertainty principle.
- Know about group velocity and phase velocity.
- Solve various numerical problems related with atomic theory.

16. How does the Heisenberg's uncertainty principle explain about the stability of the atom?

Ans: The ground state energy is the lowest possible energy with which the atom can exist and this energy is the sum of kinetic and potential energy of opposite sign. If an electron is confined in a small region, the kinetic energy becomes large and if the electron moves in large region, the potential energy will be small. Thus the ground state corresponds to the best possible compromise and this explains the stability of atom against collapse.

17. Why do not the negatively charged electrons fall into the positively charged nucleus?
OR Show that an electron cannot exist in nucleus.

Ans: The radius of an atom is of the order of 10^{-14}m. If the electron falls into the nucleus, it can be found anywhere within the diameter of the nucleus. Therefore, the maximum uncertainty in the measurement of position of electron. $\Delta x = 2 \times 10^{-14}$m. According to Heisenberg's uncertainty principle, the uncertainty between position and momentum is given by,

$$\Delta x\, \Delta p, = \frac{h}{2\pi}$$

$$\text{or,} \quad \Delta x\, m\Delta v_x = \frac{h}{2\pi}$$

$$\text{or,} \quad \Delta v_x = \frac{h}{2\pi m \Delta x} = \frac{6.6 \times 10^{-34}}{2\pi \times 9.1 \times 10^{-31} \times 2 \times 10^{-14}} = 5.7 \times 10^{9}\ \text{m/s}$$

This shows that the uncertainty in the measurement of the speed of the electron is greater than the speed of light which is impossible. This fact shows that an electron cannot exist in nucleus and hence does not fall into it.

Worked Out Examples

1. **After being excited, the electron of a Hydrogen atom eventually falls back to the ground state. This can take place is one jump or in a series of jumps, the electron falling into lower excited states before it end up in the ground state, that is n = 3. Calculate the different photon energies that may be emitted as the atom returns to the ground state.**

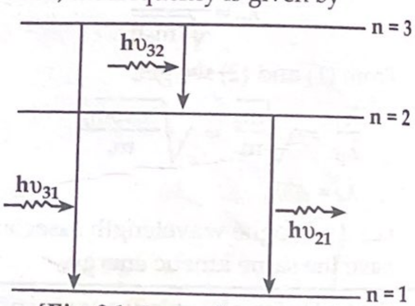
[TU Microsyllabus 2074, W;18.2]

Solution:

The possible transition lines are shown in figure 26. We know that, if an electron is initially in an allowed orbit of energy E_i and goes into another orbit of lower energy E_f. Then, the frequency is given by

$$\nu = \frac{E_i - E_f}{h}$$

$$h\nu = E_i - E_f = E_o\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$



[Fig. 26: Jump of electrons into lower excited states]

Transition from 3 to 1 states is ,

$$\therefore \quad h\nu_{31} = E_3 - E_1 = E_0\left[\frac{1}{1^2} - \frac{1}{3^2}\right]$$

$$h\nu_{31} = 13.65\ \text{eV}\left[1 - \frac{1}{9}\right] = 12.05\ \text{eV}$$

Transition from 3 to 2 states is,

$$h\nu_{32} = E_0\left[\frac{1}{2^2} - \frac{1}{3^2}\right] = 13.56\left[\frac{1}{4} - \frac{1}{9}\right] = 1.88\ \text{eV}$$

Transition from 2 to 1 state is,

$$h\nu_{21} = E_0\left[\frac{1}{1^2} - \frac{1}{2^2}\right] = 13.56\ \text{eV}\left[\frac{1}{1} - \frac{1}{4}\right] = 10.17\ \text{eV}$$

Therefore, the different photon energies that may be emitted as the atom returns to the ground state are $h\nu_{31} = 12.05$ eV, $h\nu_{32} = 1.88$ eV and $h\nu_{21} = 10.17$ eV.

2. A beam of monochromatic neutrons is incident on a KCl crystal with lattice spacing of 3.14 Å. The first order diffraction maximum is observed when the angle θ between the incident and the atomic planes is 37°. What is the kinetic energy of the neutrons?
[TU Microsyllabus 2074, W;19.1]

Solution:

We have lattice spacing (d) = 3.14Å
Order of diffraction (n) = 1
Angle of diffraction (θ) = 37°
Kinetic energy of the neutron (E_k) = ?

We know that, for crystal diffraction

$$2d \sin\theta = n\lambda$$

$$2d \sin\theta = \lambda \qquad \text{for } n = 1$$

$$\therefore \quad \lambda = 2 \times 3.14 \times \sin 37^\circ = 3.78 \text{ Å}$$

From de-Broglie's hypothesis, the momentum of the neutrons is

$$p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34} \text{ Jsec}}{3.78 \times 10^{-10} \text{ m}} = 1.75 \times 10^{-24} \text{ kg m/sec.}$$

The kinetic energy of the neutrons will be

$$E_k = \frac{1}{2} mv^2 = \frac{p^2}{2m} = \frac{(1.75 \times 10^{-24})^2}{2 \times 1.67 \times 10^{-27}} = 9.21 \times 10^{-22} \text{J}$$

Thus, the required kinetic energy $E_k = 9.21 \times 10^{-22}$ J.

3. What are the shortest and longest wavelengths of the layman series? Where, R = 1.097 × 10⁷/m.

Solution:

Let, n_1 and n_2 are the number states. We know that an expression of wavelength $\left(\frac{1}{\lambda}\right) = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

For layman series (n_1) = 1. The longest wavelength corresponds to the smallest value of n_2 which is Lyman series is 2 thus,

$$\frac{1}{\lambda_{longest}} = R\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = 1.097 \times 10^7 \left(\frac{3}{4}\right)$$

$$\text{or,} \quad \frac{1}{\lambda_L} = 0.8226 \times 10^7 \text{ m}^{-1}.$$

$$\therefore \quad \lambda_L = 1.215 \times 10^{-7} \text{ m} = 1215 \text{ Å}$$

The shortest wavelength corresponds to the largest value of n_1 that is ∞ which is

$$\frac{1}{\lambda_{shortest}} = R\left(\frac{1}{1^2} - \frac{1}{\infty^2}\right)$$

$$\text{or,} \quad \frac{1}{\lambda_s} = 1.0968 \times 10^7 \left(\frac{1}{1^2} - \frac{1}{\infty^2}\right)$$

$$\text{or,} \quad \frac{1}{\lambda_s} = 1.0968 \times 10^7 \text{ m}^{-1}$$

1-2
1-∞

Therefore $\lambda_{shortest} = 0.9117 \times 10^{-7}$m ≈ 912 Å.

Therefore, the longest and shortest wave lengths of layman series are 1215 Å and 912 Å respectively.

4. Find the energy of the neutron in unit eV whose de-Broglie's wavelength is 1Å, given mass of the neutron is 1.67 × 10⁻²⁷ kg, h = 6.62 × 10⁻³⁴ J-sec.

Solution:

We have, mass of neutron (m) = 1.67 × 10^{-27} kg
Planck's constant (h) = 6.62 × 10^{-34} J sec.
de-Broglie's wavelength (λ) = 1 Å

We know that $\lambda = \frac{h}{\sqrt{2mE_k}} \Rightarrow E_k = \frac{h^2}{2m\lambda^2}$

$$\text{or,} \quad E_k = \frac{(6.62 \times 10^{-34})^2}{2 \times 1.67 \times 10^{-27} \times (1 \times 10^{-10})^2} = 13.01 \times 10^{-21} \text{ J}$$

$$\therefore \quad E_k = \frac{13.01 \times 10^{-21}}{1.6 \times 10^{-19}} \text{ eV}$$

This is required energy of neutron in unit eV.

5. Calculate the uncertainty in the measurement of velocity of electron in H-atom of radius 1Å.

Solution:

The radius of H-atom (r) = 1 Å = 1×10^{-10} m

Uncertainty distance (Δx) = 2r = 2×10^{-10} m

We have, the uncertainty principle

$$\Delta x \,.\, \Delta p = \hbar \Rightarrow \Delta p = \frac{\hbar}{\Delta x}$$

We know that, $\Delta p = m\Delta v \Rightarrow m\Delta v = \frac{\hbar}{\Delta x}$

$$\therefore \quad \Delta v = \frac{\hbar}{m\Delta x} = \frac{h}{2\pi m \Delta x} \qquad \text{Since, } \hbar = \frac{h}{2\pi}$$

$$\Delta v = \frac{6.62 \times 10^{-34}}{2\pi \times 9.1 \times 10^{-31} \times 2 \times 10^{-11}} = 5.76 \times 10^{5}\ \text{m/sec.}$$

Thus, uncertainty in the measurement of velocity of the electron is 5.76×10^5 m/sec.

6. Calculate the shortest and the longest wavelength of the Balmer series of Hydrogen.

[TU Microsyllabus 2074, P; 18.1]

Solution:

The longest wavelength corresponding to the smallest value of n_j, which for the Balmer series is $n_j = 3$ and $n_k = 2$.

Therefore, we know that,

$$\frac{1}{\lambda_{longest}} = R\left[\frac{1}{n_k^2} - \frac{1}{n_j^2}\right]$$

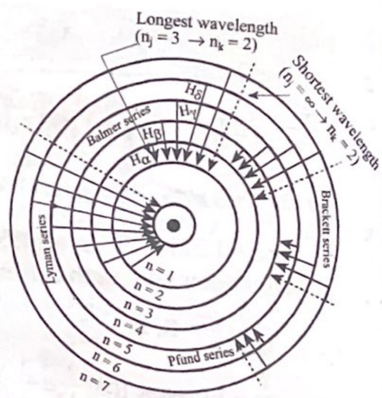
$$= R\left[\frac{1}{2^2} - \frac{1}{3^2}\right]$$

$$= 1.0968 \times 10^{7}\ \text{m}^{-1}\ [0.25 - 0.11$$

$$\frac{1}{\lambda_{longest}} = 1.523 \times 10^{6}\ \text{m}^{-1}$$

Hence, $\lambda_{longest} = \frac{1}{1.523 \times 10^{6}\ \text{m}^{-1}} = 6.54 \times 10^{-7}$ m

$\therefore \quad \lambda_{longest} = 6565$ Å.



[Fig. 27: Shortest and longest wavelength of H- atom in Balmer series]

The shortest wavelength corresponds to the largest value of n_j that is ∞.

$$\frac{1}{\lambda_{shortest}} = R\left[\frac{1}{n_k^2} - \frac{1}{n_j^2}\right]$$

$$= 10.968 \times 10^{7}\left[\frac{1}{2^2} - \frac{1}{\infty^2}\right]$$

$$= 1.0968 \times 10^{7}\ [0.25 - 0]$$

$$\frac{1}{\lambda_{shortest}} = 2742000\ \text{m}^{-1}$$

Hence, $\lambda_{shortest} = \frac{1}{2742000\ \text{m}^{-1}}$

$$= 3.6469 \times 10^{-7}$$

$\therefore \quad \lambda_{shortest} = 3646$ Å.

Hence, required shortest and longest wavelength of the Balmer series of Hydrogen are 3646 Å and 6565 Å respectively.

7. What are (a) the energy, (b) the momentum, and (c) the wavelength of photon that is emitted when a Hydrogen atom undergoes a transition from the state n = 3 to n = 1? (The momentum of the photon is given by $\frac{h\upsilon}{C}$). *[TU Microsyllabus 2074, P; 18.2, TU Exam 2074]*

Solution:

Given energy state of Hydrogen atom are $n_1 = 1$, $n_2 = 3$

Energy (E) = ?

Momentum (p) = ?

Wavelength (λ) = ?

We know that,

$$\frac{1}{\lambda} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

Where, R = Rydberg constant i.e. $R = 1.09 \times 10^7\ m^{-1}$. Then,

$$\frac{1}{\lambda} = 1.09 \times 10^7 \left[\frac{1}{1^2} - \frac{1}{3^2}\right]$$

$$\therefore \quad \lambda = 1.03 \times 10^{-7}\ m$$

Again, energy (E) = $h\frac{c}{\lambda}$

$$= 6.62 \times 10^{-34} \times \frac{3 \times 10^8}{1.03 \times 10^{-7}}$$

$$= 1.93 \times 10^{-18}\ J$$

$$\approx 12.06\ eV$$

$$\text{Momentum (p)} = \frac{h}{\lambda} = \frac{6.62 \times 10^{-34}}{1.03 \times 10^{-7}} = 6.42 \times 10^{-27}\ kg\ ms^{-1}$$

Hence, required energy, momentum and wavelength are 12.06 eV, 6.42×10^{-27} kg ms^{-1} and 1.03×10^{-7} m respectively.

8. The shortest wavelength of the Paschen series from Hydrogen is 8204 Å. From this fact, calculate the Rydberg constant. *[TU Microsyllabus 2074, P; 18.3]*

Solution:

Here is given, shortest wavelength of Paschen series $\lambda_{shortest} = 8204$ Å $= 8204 \times 10^{-10}$ m

Rydberg constant (R) = ?

We know that,

$$\frac{1}{\lambda} = R\left[\frac{1}{n_k^2} - \frac{1}{n_j^2}\right]$$

Here, shortest wavelength correspond to largest value of n_j i.e. $n_j \simeq \infty$ and for Paschen series $n_k = 3$.

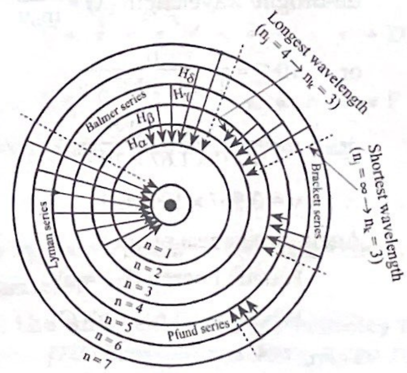
Then,

$$\text{or,} \quad \frac{1}{\lambda_{shortest}} = \left[\frac{1}{3^2} - \frac{1}{\infty^2}\right]$$

$$\text{or,} \quad \frac{1}{8204 \times 10^{-10}} = R \times \frac{1}{9}$$

$$\therefore \quad R = 10970258 = 1.097 \times 10^7\ m^{-1}$$

Hence, required Rydberg constant $1.097 \times 10^7\ m^{-1}$.



[Fig 28: Shortest and longest wavelength of H- atom in Paschen series]

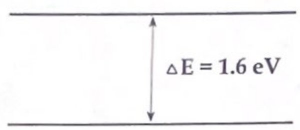
9. The ground state and the first excited-state energies of potassium atoms are – 4.3 eV and –2.7 eV respectively. If we use potassium vapour in the Franck-Hertz experience at what voltages would we see drops in the plot of current versus voltage? *[TU Microsyllabus 2074, P; 18.19]*

Solution:

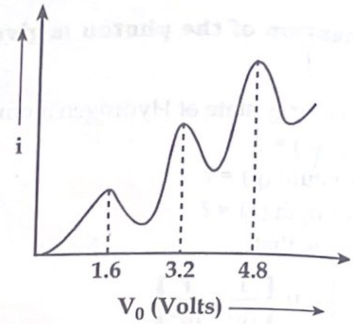
Here is given, energies corresponding to ground state and the first excited states of potassium are – 4.3eV and –2.7 eV respectively.

Here, the energy difference between the ground state and the first excited state is

$$V_0 = -2.7\text{ eV} - (-4.3\text{ eV}) = 1.6\text{ eV}$$



[Fig 29: Energy gap between states]



[Fig 30: Representation of dependency of plate current i on the accelerating voltage V_0]

As V_0 increases beyond the 1.6 eV, the current begins to increase again because, although the electrons ca and do collide inelastically and lose 1.6 eV of energy, they still have enough energy remaining to overcom the small retarding voltage. When $V_0 = 2 \times 1.6\text{ V} = 3.2\text{ V}$ or $3 \times 1.6\text{ V} = 4.8\text{ V}$ and so on, dips in the curren occur again because now electron can undergo, two, three, or more inelastic collisions with the potassium atom in each collision they lose 1.6 eV energy.

10. The de-Broglie wavelength of proton is 10^{-13} m. (a) what is the speed of the proton? (b) Through what potential difference must the proton be accelerated to acquire such a speed? ***[TU Microsyllabus 2074, P; 19.2]***

Solution:

Here is given, de-Broglie wavelength of proton (λ) = 10^{-13} m

Speed of proton (v) = ?

Potential difference (V) = ?

Mass of proton (m_p) = 1.67×10^{-27} kg

We know that,

$$\text{de-Broglie wavelength } (\lambda) = \frac{h}{m_p v}$$

$$\text{or,} \quad 10^{-13} = \frac{h}{1.67 \times 10^{-27} \times v}$$

$$\text{or,} \quad v = \frac{6.626 \times 10^{-34}}{10^{-13} \times 1.67 \times 10^{-27}}$$

$$\therefore \quad v = 3.967 \times 10^{6}\text{ ms}^{-1}$$

Again, we have a relation,

Kinetic Energy (E_k) = eV

Where, $e = 1.6 \times 10^{-19}$ C

Then,

$$\frac{1}{2} m_p v^2 = eV$$

$$V = \frac{1}{2}\frac{m_p v^2}{e}$$

$$= \frac{1}{2}\frac{(1.67 \times 10^{-27}) \times (3.967 \times 10^{6})^2}{1.6 \times 10^{-19}} = 8.2015 \times 10^4$$

$$\therefore \quad V = 8.20 \times 10^{4}\text{ Volt}$$

Hence, required speed of proton and accelerating p.d. are $3.967 \times 10^{6}\text{ ms}^{-1}$ and 8.20×10^{4} volt respectively.

11. An α-particle is emitted from a nucleus with an energy of 5 MeV (5×10^6 eV). Calculate the wavelength of an α-particle with such energy and compare it with the size of the emitting nucleus that has a radius of 8×10^{-15} m. *[TU Microsyllabus 2074, P; 19.7]*

Solution:

Here is given, energy of α-particle (E_k) = 5 MeV = $5 \times 10^6 \times 1.6 \times 10^{-19}$J

Wavelength of α-particle (λ_α) = ?

Radius of the nucleus (r) = 8×10^{-15} m

We know that, from de-Broglie hypothesis

$$\lambda_\alpha = \frac{h}{\sqrt{2m_\alpha E_k}}$$

$$= \frac{6.626 \times 10^{-34}}{\sqrt{2 \times 4 \times 1.67 \times 10^{-27} \times 5 \times 10^6 \times 1.6 \times 10^{-19}}}$$ Since, α-particle ($_2He^4$) consist of 2n and 2p = 4 m_p

∴ $\lambda_\alpha = 6.409 \times 10^{-15}$ m

Hence, wavelength of an α-particle i.e. 6.409×10^{-15} m is less than that of radius i.e. 8×10^{-15} m.

12. A neutron spectroscopy a beam of mono energetic neutrons is obtained by reflecting reactor neutrons from a beryllium crystal. If separation between the atomic planes of the beryllium crystal is 0.732 Å, what is the angle between the incident neutron beam and the atomic planes that will yield a monochromatic beam of neutrons of wavelength 0.1 Å. *[TU Microsyllabus 2074, P; 19.11]*

Solution:

Here is given, Separation between the atomic plane (d) = 0.732 Å = 0.732×10^{-10} m

Wavelength of monochromatic beam of neutrons (λ) = 0.1 Å = 0.1×10^{-10} m

Angle between the incident neutron beam and the atomic planes (θ) = ?

From Brogg's law,

$$2d \sin \theta = n\lambda$$

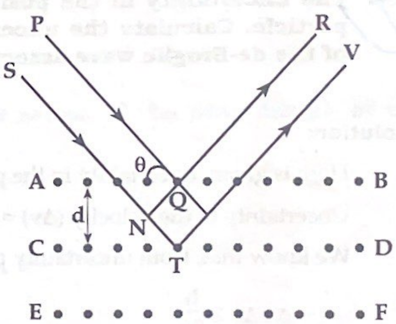
or, $\sin \theta = \frac{n\lambda}{2d}$ [Since, n = 1, for first order]

or, $\theta = \sin^{-1}\left(\frac{\lambda}{2d}\right)$

or, $\theta = \sin^{-1}\left(\frac{0.1 \times 10^{-10}}{2 \times 0.732 \times 10^{-10}}\right)$

∴ θ = 3.92°

Hence, required angle between the incident neutron bean and the atomic plane is 3.92°.



[Fig. 31: Bragg's diffraction]

13. A small particle of mass 10^{-6} g moves along the x-axis; its speed is uncertain by 10^{-6} ms^{-1}.

a. What is the uncertainty in the x-coordinate of the particle?

b. Repeat the calculation for an electron assuming that the uncertainty in its velocity is also 10^{-6} ms^{-1}. *[TU Microsyllabus 2074, P; 19.16]*

Solution:

Here, given mass of small particle (m) = 10^{-6} g = $10^{-6} \times 10^{-3}$ kg

Speed of particle (v) = 10^{-6} ms^{-1}

Uncertainty in the x-coordinate of the particle (Δx) = ?

Uncertainty in velocity (Δv) = 10^{-6} ms^{-1}

We know that, from Heisenberg's uncertainty principle,

$$\Delta x \, \Delta p \geq \hbar$$

or, $\Delta x \, \Delta p = \frac{h}{2\pi}$

a. Here, $\Delta p = m_{particle}\Delta v$

$$\therefore \quad \Delta x = \frac{h}{2\pi\Delta p}$$

$$= \frac{h}{2\pi m\Delta v}$$

$$= \frac{6.626 \times 10^{-34}}{2\pi \times 10^{-9} \times 10^{-6}}$$

$$= 1.05 \times 10^{-19}\text{ m}$$

$$\therefore \quad \Delta x = 1.05 \times 10^{-19}\text{ m}$$

b. Mass of electron $m_e = 9.1 \times 10^{-31}$kg

Then, uncertainty in x-coordinate or position of electron

$$\Delta x = \frac{h}{2\pi\Delta p}$$

$$= \frac{h}{2\pi m\Delta v}$$

$$= \frac{6.626 \times 10^{-34}}{2\pi \times 9.1 \times 10^{-31} \times 10^{-6}} = 115.768$$

$$\therefore \quad \Delta x = 115.768 \approx 116\text{ m.}$$

Hence, uncertainty in x-coordinate for a particle and electron are 1.05×10^{-19} m and 116 m respectively.

14. The uncertainty in the position of a particle is equal to the de-Broglie wavelength of the particle. Calculate the uncertainty in the velocity of the particle in terms of the velocity of the de-Broglie wave associated with the particle.

[TU Microsyllabus 2074, P;19.19, TU Exam 2074]

Solution:

Here is given, uncertainty in the position of a particle say Δx = de-Broglie wavelength of the particle (λ)

Uncertainty in the velocity (Δv) = ?

We know that, from uncertainty principle,

$$\Delta x\,\Delta p \geq \frac{h}{2\pi}$$

$$\text{or,} \quad \Delta x = \frac{h}{2\pi\Delta p}$$

$$= \frac{h}{2\pi m\Delta v} \quad \text{... (1)}$$

Again, from de-Broglie hypothesis,

$$\lambda = \frac{h}{mv} \quad \text{... (2)}$$

According to question, from equation (1) and (2), we get

$$\frac{h}{2\pi m\Delta v} = \frac{h}{mv}$$

$$\therefore \quad \Delta v = \frac{v_{wave}}{2\pi}$$

Hence, uncertainty in velocity is equal to the $\frac{1}{2\pi}$ times velocity of the de-Broglie wave.